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Probability and Applied Statistics

CSCI-3327

Normal, Gamma, and Beta Probability Distributions

**INTRODUCTION**

A probability distribution is a function that calculates the probability of a variable holding different values. In other words, it gives the probability of certain events occurring in an experiment. If *Y* is a random variable, the distribution function of *Y*, denoted *F(y)*, is for . In addition to the uniform probability distribution where all events are likely to occur, there are normal, beta, and gamma probability distributions.

**NORMAL PROBABILITY DISTRIBUTION**

Diagram

Description automatically generated The normal probability distribution is the most widely used continuous distribution along with its well-known bell curve. When looking at the graph, the probability is represented by the area underneath the curve which equals 1. The mean is located in the center, denoted by the parameter , and its spread or delta is denoted by .

A random variable *Y* is said to have a normal probability distribution if and only if, for and , the density function of *Y* is

.

If *Y* is a normally distributed random variable with parameters and , then and , where *E(Y)* is a expected value of *Y* and *V(Y)* is the variance.

*z* is the distance from the mean of a normal distribution expressed in units of standard deviation. Therefore,

A similar formula can transform a normal random variable *Y* to a standard normal random variable *Z* by using the relationship

*Z* locates a point measured from the mean of a normal random variable, with the distance expressed in units of the standard deviation of the original normal random variable. Therefore, the mean of *Z* is 0 and the standard deviation is 1.

**Example:** The weekly amount of money spent on maintenance and repairs by a company was observed, over a long period of time, to be approximately normally distributed with mean $400 and standard deviation $20. If $450 is budgeted for next week, what is the probability that the actual costs will exceed the budgeted amount?

(from the attached table)

**GAMMA PROBABILITY DISTRIBUTION**

Shape, rectangle

Description automatically generated The gamma probability distribution is right-skewed function where random variables are always nonnegative. Most of the values underneath the curve are near 0. Gamma distributions are used for things that have a natural value of 0 such as elapsed time and time between events.

A random variable *Y* is said to have a gamma distribution with parameters and if and only if the density function of *Y* is

where

is the gamma function. is sometimes called the shape parameter because the shape of the gamma density differs for different values of . is called the scale parameter since multiplying a gamma-distributed random variable by a constant produces a random variable that has a different scale, but also has the same shape of .

Also, if *Y* has a gamma distribution with parameters and , then

and

A special gamma distribution is called the chi-square distribution. If *v* is a positive integer, a random variable *Y* is said to have a chi-square distribution with *v* degrees of freedom if and only if *Y* is a gamma-distributed random variable with parameters and .

If *Y* is a chi-square random variable with *v* degrees of freedom, then

and

Another special gamma distribution, where , is the exponential distribution. If *Y* is said to have an exponential distribution with parameter if and only if the density function of *Y* is

If *Y* is an exponential random variable with parameter , then

and

**Example:** The magnitude of earthquakes recorded in a region of North America can be modeled as having an exponential distribution with mean 2.4, as measured on the Richter scale. Find the probability that an earthquake striking this region will exceed 3.0 on the Richter scale.

**BETA PROBABILITY DISTRIBUTION**

The beta probability distribution is used to model proportions, such as a proportion of time. Similar to gamma distributions, the beta density function has two parameters and the easiest way to calculate probability using them is with mathematical software.

A random variable *Y* is said to have a beta probability distribution with parameters and if and only if the density function of *Y* is

where

The graphs of the functions are different for various values of the parameters and .

If *Y* is a beta-distributed random variable with parameters and , then

and

**Example:** The percentage of impurities per batch in a chemical product is a random variable Y with density function

A batch with more than 40% impurities cannot be sold.

Integrate the density directly to determine the probability that a randomly selected batch cannot be sold because of excessive impurities.

Table

Description automatically generated